JOURNAL OF RESEARCH ON APPLIED GEOPHYSICS

(JRAG)

Shahrood University of Technology

2026, VOL 12, NO 1

(DOI): 10.22044/JRAG.2026.17251.1379

2D gravity data inversion using active constraint balancing method and the Lanczos algorithm - A case study: Esfandar iron ore mine

Meysam Moghadasi[1]; Ali Nejati Kalateh[2*]; Mohamad Rezaei[3]; Mehrdad Soleimani Monfared[4]

1. PhD Student, Faculty of Mining, Petroleum and Geophysics, Shahrood University of Technology, Shahrood, Iran.
2. Associate Professor, Faculty of Mining, Petroleum and Geophysics, Shahrood University of Technology, Shahrood, Iran.
3. Assistant Professor, Malayer university, Malayer, Iran.
4. Associate Professor, Faculty of Mining, Petroleum and Geophysics, Shahrood University of Technology, Shahrood, Iran.

Received: 1 December 2025 ; Accepted: 26 January 2026

Corresponding author: nejati@shahroodut.ac.ir

Keywords

Inverse modeling
Gravity inversion
Regularization parameter
Esfandar

Extended Abstract

Summary

Geophysical methods, by providing indirect measurements of subsurface properties, play a vital role in inferring the existence of subsurface reserves. The inversion of gravity data is one of the most critical steps in the interpretation of these datasets. Its objective is to estimate the density distribution of an unknown subsurface model using data measured at the earth surface. The primary challenge in the inversion of gravity data is the inherent non-uniqueness of the solution. Linear inversion of gravity data typically constitutes an underdetermined and ill-posed problem. In this research, active constraint balancing (ACB) method is employed to determine the optimal regularization parameter for two-dimensional (2D) inversion of gravity data, solved using the Lanczos bidiagonalization (LSQR) algorithm. To this end, an algorithm was developed to compute the optimal regularization parameter for the inversion. For performance evaluation and validation, the algorithm was first applied to synthetic data, and subsequently, to real gravity data from the Esfandar iron ore mine, located in Yazd Province, Iran.

Introduction

Geophysical data inversion is one of the most important steps in the quantitative interpretation of geophysical data. In the present study, the results obtained from gravity data inversion provide useful information about subsurface structures, density contrasts, and depth of subsurface masses (Lee and Oldenburg, 1998). Gravity data inversion involves many complexities. The main problem with gravity data inversion problems is the inherent non-uniqueness in the answers to these problems. Gaussian theory from a given set of gravity data, abundance distributions of subsurface sources can be inferred (Blakely, 1996). By considering the initial information for an inverse problem, the non-uniqueness of the final answer to the problem can be overcome to some extent (Farquharson, 2008). Martinez et al. (2010) developed a smooth inversion method for inverting gravity gradiometric data based on the Lee and Oldenberg inversion method (Martinez et al., 2010). Rezaei et al. (2017) developed a data space inversion method with dispersion constraints for 3D inversion of gravity data with upper and lower bound constraints on physical parameters (Rezaei et al., 2017). Moghadasi et al. (2018) used the ACB method to calculate the regularization parameter for inverting gravity data in the Kamagi chromite mine (Moghadasi et al., 2018). Abedi et al. (2013) applied the Lanczos bidiagonalization algorithm for 3D inversion of magnetism data with zero-order Tikhonov condition. They showed that this algorithm achieves the solution faster than the conjugate gradient algorithm (Abedi et al., 2013).

Moghadasi et al. (2019) developed an algorithm for calculating the regularization parameter using the ACB method and three-dimensional inversion of gravity data. Moorkamp (2021) presented a method for simultaneous inversion of gravity and magnetotelluric data on the Ernst-Henry IOCG repository using different information constraints. Vatankhah et al. (2022) presented a method for large-scale simultaneous centralized inversion of gravity and magnetometry data using the Grarmain constraint. This method provides good similarity between reconstructed models, while implementing the Gramain constraint in the space of weighted parameters provides good correlation for the weighted parameters. Zhdanov et al. (2022) presented a method for collaborative inversion of airborne gravity and magnetometric data using the joint minimum entropy constraint.

Methodology and Approaches

In inverse modeling, model parameters are obtained by minimizing the Tikhonov objective function, which is less ill-posed (Tikhonov et al., 1977). For this purpose, Tikhonov's objective function is defined as follows (Lee and oldenburg, 2003):

$$\min_{\rho}(|V_d(G\rho - d_{obs})|_2^2 + \beta|D\rho|_2^2) \tag{1}$$

In equation, $\Phi_d = |V_d(G\rho - d_{obs})|_2^2$ is the weighted data misfit function, $\Phi_\rho(\rho) = |D\rho|_2^2$ is the Tikhonov stabilizing function, and ρ is the model parameter. Moreover, β is a regularization parameter that balances the data misfit function and the Tikhonov stabilizing function. The objective function specified in equation (1) is transformed into the final form of equation (2) to be minimized:

$$\emptyset(\rho) = \arg\min\{|W_d(G\rho - d_{obs})|_2^2 + \beta|W_{depth}D\rho|_2^2\} \tag{2}$$

By treating the Lagrange multiplier as a spatially varying quantity within the regularization term, it becomes possible to appropriately balance the regularization applied in the inversion process. The spatial distribution of the Lagrange multipliers (regularization parameters) is determined through analysis of the parameter resolution matrix and the Backus–Gilbert spread function. In the ACB approach for estimating the regularization parameter, the resolution matrix R must be computed. This parameter resolution matrix R can be derived during the inversion procedure as the product of the pseudo-inverse and the kernel matrix G.

$$R = G^+G \tag{3}$$

The spread function, which accounts for the inherent degree of how much the ith model parameter is not resolvable, is defined as;

$$SP_i = \sum_{j=1}^{N}\left(w_{ij}\left(1 - s_{ij}\right)R_{ij}\right)^2 \tag{4}$$

Here, M denotes the total number of model parameters involved in the inversion. The term w_{ij} represents a weighting coefficient determined by the spatial separation between the ith and jth model parameters, while c_{ij} is a factor that indicates whether the applied constraint or regularization affects the ith parameter and its neighboring parameters. Put differently, the spread function defined in this formulation corresponds to the sum of the squared, spatially weighted dispersion of the ith model parameter compared to all model parameters, excluding those to which a smoothness constraint is applied. Within this framework, the regularization parameter $\lambda(x, z)$ is assigned using values obtained through log-linear interpolation.

$$\log(\lambda_i) = \log(\lambda_{\min}) + \frac{\log(\lambda_{\max}) - \log(\lambda_{\min})}{\log(SP_{\max}) - \log(SP_{\min})} \times \{\log(SP_i) - \log(SP_{\min})\} \tag{5}$$

In this formulation, SP_{min} and SP_{max} denote the minimum and maximum values of the spread function, respectively, while $\lambda_{\min}$ and $\lambda_{\max}$ respectively represent the lower and upper bounds of the regularization parameter $\lambda(x, z)$, which must be specified by the user. This strategy enables the automatic assignment of smaller values of $\lambda(x, z)$ to model parameters that are more well resolved—corresponding to smaller values of the spread function during the inversion process—and larger values otherwise. Users may define these minimum and maximum regularization bounds through the variables LambdaMin and LambdaMax. To achieve the desired target, a dedicated algorithm has been developed to estimate this parameter.

Results and Conclusions

The Esfandar iron ore mine is located approximately 53 km southeast of the city of Abarkuh in Yazd Province. From a geological perspective, the study area forms part of the Sanandaj–Sirjan Zone and is located adjacent to Mount Hambast, to the northwest of the Abarkuh playa. Outcrops of granitic intrusions are observable within the area, particularly at the intrusive contact zones. Investigation of the lithological units indicates that the intrusive body and iron-bearing apophyses have been emplaced as numerous patches into carbonate horizons, with mineralization occurring along their contacts. This interpretation is supported by the widespread development of epidotization and magnetite mineralization within the epidote-altered units. In the mining area, due to fault activity and the presence of intrusive bodies, ore mineralization is manifested in several discrete outcrops, a feature that is clearly reflected in the magnetometric maps. The presence of multiple fault systems in the area has facilitated the migration of mineralizing fluids along these structures, leading to ore formation in structurally favorable zones. Various geophysical methods have been conducted at this mine, resulting in the generation of valuable information layers, including magnetic, gravity, and magnetotelluric data. In this study, a gravity profile is utilized to evaluate the performance of the proposed inversion algorithm. Moreover, the availability of magnetic and magnetotelluric datasets makes this case study useful and practical for assessing both individual and joint inversion algorithms. A comparison between the results obtained from the magnetotelluric and gravity studies indicates a satisfactory level of consistency and agreement in the geometric and depth estimations produced by the proposed algorithm. The profile used in this study is oriented in the north–south direction and has a length of 1200 m. The spacing between gravity stations along this profile is 10 m. Considering the sampling interval, the dimensions of each block are 10 × 10 m. Based on the inversion results of the real data, the presence of an anomalous body with a density contrast of 0.05 g/cm^3 is evident. The depth extent estimated from the gravity inversion results indicates that the anomaly extends to an approximate depth of 300 m. Given the relatively large estimated depth obtained from this algorithm, incorporating other geophysical datasets, such as magnetic and magnetotelluric data, within a joint inversion framework can improve the accuracy of both the depth estimation and the geometric characterization of the anomalous body, and yield more reliable and informative results.

In this research, the challenge of choosing the optimal regularization parameter in 2D inversion of gravity data was investigated. For this purpose, an algorithm based on ACB method and Lanczos bidiagonalization algorithm (LSQR) was developed. The evaluation of the algorithm on a synthetic model and real data of Esfandar iron ore mine showed that the proposed method can establish a suitable balance between the stability of the inversion and the resolution of the model, and compared to traditional methods with fixed parameters, it achieves more reliable and accurate results of subsurface structures. According to the results of the studies of other geophysical methods such as the magnetotelluric method, a very favorable match in terms of geometric structure between the gravity model obtained from the proposed algorithm and the magnetotelluric model is observed.